GLUEBALL SPECTRUM AND REGGE TRAJECTORY FROM SUPERGRAVITY

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Brief review of the status of the glueball spectrum in the deformed conifold background. Talk based on work done in collaboration with with R. Herández and X. Amador.

1. Introduction

The formulation of a gauge/string duality [1] provides a new framework to study confining phenomena. The duality was originally formulated between type IIB string theory propagating in Anti de Sitter (AdS) space and a conformal field theory (CFT). Nowadays several supergravity duals to confining gauge theories are known [2,3]. Some of this backgrounds are deformations of the original AdS/CFT duality. This is the case of AdS blackhole backgrounds dual to a finite temperature field theory. The spectrum in these backgrounds has been thoroughly studied by Brower, Mathur and Tan[4]. In the present work we focus on the Klebanov-Strassler (KS) IIB supergravity solution [3]. This background describes N regular and M fractional D3 branes on a deformed conifold space; it is conjectured to be dual to a cascading gauge theory with $SU(M+N)\times SU(N)$ gauge group in the ultraviolet and flows to $SU(M)$ Super-Yang Mills in the infrared. The glueball spectrum for 0^{++} and 1^{--} in the KS background was obtained in Ref.5 and the spin 2 glueball mass was recently obtained in Ref.6. We will review this results and address possible comparisons with lattice results. In particular, features of the Regge trajectory as obtained from this spectrum.

Regge theory successfully describes a large quantity of experimental data. It predicts that composite particles of a given set of quantum numbers, different only in their spin, will lie on a linear trajectory

$$J = \alpha_0 + \alpha^{'} t$$

where J is the spin and t is the mass squared. Regge theory treats the strong interaction as the exchange of a complete trajectory of particles. With the inclusion of a soft pomeron, this approach successfully describes the high energy scattering of hadrons. It is thus interesting to explore Regge trajectories from the gauge/ gravity

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duality. We show that the KS background predicts a *linear* glueball trajectory, $J = -2.29 + 0.23\, t$. Unlike the scenario where glueball masses are identified with classical solutions of folded strings, here there is no *a priori* reason for the Regge trajectory to be linear. That it turns out to be so is remarkable.

2. The Deformed Conifold

The Klebanov-Strassler background [3] is a solution to type II B supergravity equations with non-zero three and five forms and a constant dilaton. The KS solution is rich in interesting physical phenomena; exhibits confinement, chiral symmetry breaking, dimensional transmutation, domain walls etc. We will review some aspects of the KS background necessary for the next sections .

The solution describes N regular and M fractional D3 branes on a deformed conifold and is given by,

$$ds^2_{10} = h^{-1/2}(\tau) dx_n dx_n + h^{1/2}(\tau) ds_6^2 \tag{1}$$

where

$$h(\tau) = (g_s M\alpha')^2 2^{2/3} \epsilon^{-8/3} I(\tau) \equiv \int_\tau^\infty dx\ \frac{(x\coth x - 1)(\sinh(2x) - 2x)^{1/3}}{\sinh^2 x},$$

$$\mathcal{F}_5 = B_2 \wedge F_3 = \frac{g_s M^2 (\alpha')^2}{4} l(\tau) g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5$$

$$F_3 = \frac{M\alpha'}{2} \left\{ g^5 \wedge g^3 \wedge g^4 + d[F(\tau)(g^1 \wedge g^3 + g^2 \wedge g^4)] \right\}$$

$$H_3 = dB_2 = \frac{g_s M\alpha'}{2} \left[d\tau \wedge (f' g^1 \wedge g^2 + k' g^3 \wedge g^4) + \frac{1}{2}(k - f) g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \right].$$

The precise form of the functions defining the deformed conifold metric, ds^2_6, the three-form, F_3, and five-form,F_5, can be found in Ref.3.

In the infrared region , $\tau \to 0$, the metric becomes

$$\begin{aligned} ds^2_{10} \to & \frac{\varepsilon^{4/3}}{2^{1/3} a_0^{1/2} g_s M\alpha'} dx_n dx_n + a_0^{1/2} 6^{-1/3} (g_s M\alpha') \Big\{ \frac{1}{2} d\tau^2 + \frac{1}{2}(g^5)^2 \\ & + (g^3)^2 + (g^4)^2 + \frac{1}{4}\tau^2 [(g^1)^2 + (g^2)^2] \Big\} . \end{aligned} \tag{2}$$

The parameter $\epsilon^{2/3}$ has dimensions of length and measures the deformation of the conifold. From the IR metric (2) we see that $\epsilon^{2/3}$ also sets the dynamically generated 4-d mass scale $\frac{\epsilon^{2/3}}{\alpha' \sqrt{g_s M}}$ and the glueball masses scale $m_{glueball} \sim \frac{\epsilon^{2/3}}{\alpha' g_s M}$.

3. Glueball Masses and Regge Trajectory

3.1. O^{++} *and* 1^{--}

The simplest case to consider is the 0^{++} which is dual to fluctuations of the dilaton field $\tilde{\phi}$. After linearizing the equations of motion it is standard procedure to make an expansion in harmonics on the angular part of the transverse space, $\tilde{\phi} = \Sigma_I \Phi^I(\bar{x},\tau)Y^I(\bar{\theta})$, where $\bar{x}$ is shorthand for x_1, x_2, x_3, x_4 coordinates in Minkowski space, τ ir a radial coordinate and $\bar{\theta}$ denotes all the angular coordinates. Collecting the equations for scalar harmonics we find that the dilaton fluctuation decouples from the other fluctuations, the relevant equation is

$$\Box\tilde{\Phi}(\bar{x},\tau) = 0.$$

The next step is to expand in plane waves $\tilde{\phi}(\tau,\bar{x}) = e^{i\mathbf{k}\cdot\bar{x}} f(\tau)$. Using the ten dimensional metric (1) we get,

$$3.2^{1/3}\frac{d}{d\tau}\left[(\sinh(2\tau)-2\tau)^{2/3}\frac{df(\tau)}{d\tau}\right] - (k^2\epsilon^{4/3})\sinh^2(\tau)h(\tau)f((\tau) = 0. \tag{3}$$

where each mode has mass $m^2 = -k^2$. The spin zero glueball masses are identified with the values of k^2 for which eq.(3) has a normalizable solution. Solving this eigenvalue problem numerically we get,

$$m^2(0^{++}) = 9.78, \qquad m^2(0^{++*}) = 33.17$$

where the masses are measured in units of the conifold deformation $\epsilon^{\frac{4}{3}}$.The same procedure can be applied for the 1^{--} that is identified with a gauge field in the supergravity side. In this case the equations do not decouple and we have to solve a system of equations (see Ref. 5 for details). We obtain, $m^2(1^{--}) = 14.05$, $m^2(1^{--*}) = 42.90$.

3.2. *Spin Two Glueball*

Finding the spin two glueball mass is,technically, more challenging since we have to solve for fluctuations of the metric. Consider the metric,

$$g_{MN} = g^{KS}_{MN} + h_{MN}$$

where g^{KS}_{MN} is the Klebanov-Strassler background metric (eq. 1) and h_{MN} denotes fluctuations around this background. In the present case we are interested in infrared phenomena *i.e.* in the $\tau \to 0$ region. In this region the angular part of the deformed conifold behaves as an S^3 that remains finite -with radius of order $g_s M$ at $\tau = 0-$ and an S^2 which shrinks like τ^2 . Thus, for small τ it is appropriate to expand in spherical harmonics on the S^3.

After introducing the expansion in harmonics in the IIB supergravity equations and keeping in mind that we are interested in fluctuations on the four dimensional

space transverse to the deformed conifold we find that the linearized equation for the fluctuations is

$$-\frac{1}{2}\nabla^\lambda\nabla_\lambda h_{ij}(\tau,\bar{x}) + \frac{1}{2}\nabla^l\nabla_i h_{lj}(\tau,\bar{x}) + \frac{1}{2}\nabla^l\nabla_j h_{li}(\tau,\bar{x}) =$$
$$\left(\frac{g_s^2}{96}\left(\frac{1}{5}\star\mathcal{F}_5^{KS}\cdot\star\mathcal{F}_5^{KS}\right) - \frac{g_s^2}{48}H_3^{KS}\cdot H_3^{KS} - \frac{1}{48}F_3^{KS}\cdot F_3^{KS}\right)h_{ij}(\tau,\bar{x}) \qquad (4)$$

where the covariant derivative is with respect to the full KS background (1). Expanding in plane waves,

$$g_{ij}(\tau,\bar{x}) = h_{ij}(\tau)e^{i\mathbf{k}x},$$

a mode of momentum k has a mass $\mathbf{k}^2 = -m^2$. The spin 2 representation of the $SO(3)$ symmetry in x_2, x_3, x_4 is a symmetric traceless tensor. Choosing a gauge $g_{1i}(\tau,\bar{x}) = 0$, the fluctuation $h_{ij}(\tau)$ ($i,j = 1,2,3,4$) has five independent components corresponding to the five polarizations of the 2^{++}. As expected, all five satisfy the same equation of motion and are thus degenerate. Denoting $g(\tau) \equiv h_{22}(\tau) = h_{33}(\tau) = h_{23}(\tau) = h_{24}(\tau) = h_{34}(\tau)$ we obtain (see Appendix for details) from (4),

$$\frac{d^2}{d\tau^2}g(\tau) + A(\tau)\frac{d}{d\tau}g(\tau) + \left(B(\tau) - \frac{g_s^2\alpha^2 M^2}{2^{1/3}\epsilon^{4/3}}I(\tau)G_{55}(\tau)\mathbf{k}^2\right)g(\tau) = 0 \qquad (5)$$

where,

$$A(\tau) = \frac{d\ln(G_{99}(\tau))}{d\tau} + \frac{d\ln(G_{77}(\tau))}{d\tau} + \frac{d\ln I(\tau)}{d\tau}$$

and

$$B(\tau) = \frac{-2^{(1/3)}(1-F(\tau))^2}{8I(\tau)G_{77}(\tau)^2} - (\frac{dk(\tau)}{d\tau})^2 - \frac{2^{1/3}(k(\tau)-f(\tau))^2}{16G_{77}(\tau)G_{99}(\tau)} + 4(\frac{dF(\tau)}{d\tau})^2 -$$
$$\frac{2^{1/3}}{8I(\tau)G_{99}(\tau)^2}((\frac{df(\tau)}{d\tau})^2 + F(\tau)^2) + \frac{1}{4I(\tau)^2}\left(\frac{dI(\tau)}{d\tau}\right)^2 - \frac{2^{5/3}l(\tau)^2}{I(\tau)^2K(\tau)^4(\cosh^2\tau - 1)^2}$$

$G_{77}(\tau)$ and $G_{99}(\tau)$ are redefinitions of the background metric $g_{MN}^{KS}(\tau)$ such that $g_{ii}^{KS}(\tau) = h(\tau)^{-1/2}G_{ii}(\tau)$ for $i = 1...4$ and $g_{\mu\mu}^{KS}(\tau) = h(\tau)^{1/2}\epsilon^{4/3}G_{\mu\mu}(\tau)$ for $\mu = 5...10$, they do not contain dimensionfull quantities. Spin two glueball masses are identified with the values of $\mathbf{k}^2$ for which there is a solution of (5) with boundary conditions that will ensure the solution is renormalizable.

3.3. *Results*

A spreviously stated, equations (3) and (5) are eigenvalue problems. They can be solved exactly by a variety of numerical methods. The boundary conditions at infinity are found by demanding normalizability of the states. We used a "shooting technique" to find the eigenvalues. This method is very accurate for low lying states

but requires an initial guess for the eigenvalue. We used as initial guess the value found from a WKB approximation. Equation (5) is particulaarly sensitive to accumulation of numerical error due to the combination of hyperbolic functions involved in the coefficients. In order to overcome this difficulty we calculate $A(\tau)$and $B(\tau)$ with 20 digits of precision . With this technique we find a very stable eigenvalue for $m^2(2^{++}) = 18.33$ We collect the results for the lowest lying 0^{++}, 1^{--} and 2^{++} in Table 1. The Chew-Frautschi plot for the glueball trajectory obtained with these

Table 1. Glueball masses

State	0^{++}	1^{--}	2^{++}
$Mass^2/\epsilon^{4/3}$	9.78	14.05	18.33

values is shown in Figure 2. It is remarkable that the three states lie on a straight line. For large quantum numbers it is known that glueballs can be identified with spinning folded closed strings. In that approach a linear Regge trajectory is no surprise since it is built in the formalism. But in the present framework, where we identify masses with eigenvalues of equations of motion, there is no *a priori* reason for the eigenvalues to lie on a straight line. The fact that it is so is remarkable. The glueball Regge trajectory obtained from the KS model is $J = -2.2 + 0.23t$. We also

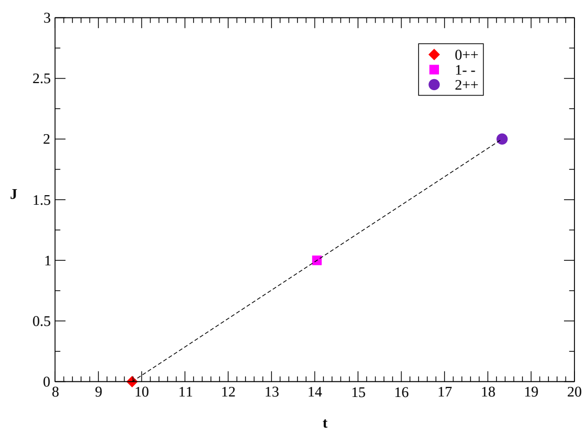

Fig. 1. Glueball trajectory. The mass squared, t, is measured in units of $\frac{\epsilon^{4/3}}{g_s^2 \alpha'^2 M^2}$.

compare glueball mass ratios with lattice results. Lattice data for $D = 4$ is not as abundant as for $D = 3$. Lucini and Teper explored the $N \to \infty$ limit of $SU(N)$ Yang-Mills theory in four dimensions [7]. We present their results in Table 2 and show that the agreement with supergravity results obtained from the KS background is

within one standard deviation. It is interesting to include in the comparison lattice results for $SU(3)$ in $D = 4$ [8].

Table 2. Glueball mass ratios calculated in the KS model and in two lattice simulations; $SU(\infty)$ and $SU(3)$.

	KS model	Lattice $SU(\infty)$	Lattice QCD_4
$m(0^{++*})/m(0^{++})$	1.84	1.91(17)	1.79(6) [a]
$m(2^{++})/m(0^{++})$	1.37	1.46(11)	1.39(4)

Note that the trajectory found here using a supergravity approach does not reproduce what is expected for a Pomeron trajectory. In our opinion it is too premature to make direct comparisons with experiimantal values for the Pomeron Regge trajectory. We consider that understanding Regge trajectories from supergravity is an important issue; the good agreement of mass ratios with lattice results and the fact that the trajectory obtained is linear show that we are making progress in that direction.

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Dedicated to the memory of Iciar Isusi
Tarma, 1967 - Lima, 2002, Perú.

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